

# Quantum mechanical approach to decoherence and relaxation generated by fluctuating environment

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We consider an electrostatic qubit, interacting with fluctuating charge of a single-electron transistor (SET) in the framework of an exactly solvable model. The SET plays role of an environment affecting the qubits' parameters in a controllable way. We derive the rate equations describing the dynamics of the entire system for an arbitrary qubit-SET coupling. Solving these equations we obtain decoherence and relaxation rates of the qubit, as well as the spectral density of qubit parameters' fluctuations. We found that in a weak-coupling regime decoherence and relaxation rates are directly related to the spectral density taken at either zero or Rabi frequency, depending on which qubit parameter is fluctuating. In the latter case our result coincides with that of the spin-boson model in the weak-coupling limit, despite different origin of the fluctuations. We show that this relation holds also in the presence of weak backaction of the qubit on the environment. In case of strong backaction such a simple relationship no longer holds, even if qubit-SET coupling is small. It does not hold also in the strong-coupling regime, even in the absence of the backaction. In addition, we found that our model predicts localization of the qubit in the strong-coupling regime, resembling that in the spin-boson model.

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## I. INTRODUCTION

The influence of environment on a single quantum system is the issue of crucial importance in quantum information science. It is mainly associated with decoherence, or dephasing, which transforms any pure state of a quantum system into a statistical mixture. Despite a large body of theoretical work devoted to decoherence, its mechanism has not been clarified enough. For instance, how decoherence is related to environmental noise, in particular, in the presence of backaction of the system on the environment (quantum measurements). Moreover, decoherence is often intermixed with relaxation. Although each of them represents an irreversible process, decoherence and relaxation affect quantum systems in quite different ways.

In order to establish a relation between the fluctuation spectrum and decoherence and relaxation rates one needs a model that describes the effects of decoherence and relaxation in a consistent quantum mechanical way. An obvious candidate is the spin-boson model<sup>1,2</sup> which represents the environment as a bath of harmonic oscillators at equilibrium, where the fluctuations obey Gaussian statistics.<sup>3</sup> Despite its apparent simplicity, the spin-boson model cannot be solved exactly.<sup>2</sup> Also, it is hard to manipulate the fluctuation spectrum in the framework of this model. In addition, mesoscopic structures may couple only to a few isolated fluctuators, such as spins, local currents, background charge fluctuations, etc. This would require models of the environment, different from the spin-boson model (see, for instance, Refs. 4–12). In general, the environment can be out of equilibrium, like a steady-state fluctuating current, interacting with the qubit.<sup>13–16</sup> This, for instance, takes place in the continuous measurement (monitoring) of quantum systems<sup>17</sup> and in the

“control dephasing” experiments.<sup>18–20</sup> All these types of non-Gaussian and nonequilibrium environments recently attracted a great deal of attention.<sup>21</sup>

In this paper we consider an electrostatic qubit, which can be viewed as a generic example of two-state systems. It is realized by an electron trapped in coupled quantum dots,<sup>22–24</sup> Fig. 1. Here  $E_1$  and  $E_2$  denote energies of the electron states in each of the dots and  $\Omega_0$  is a coupling between these states. It is reasonable to assume that the decoherence of a qubit is associated with fluctuations of the qubit parameters,  $E_{1,2}$  and  $\Omega_0$ , generated by the environment. Indeed, a stochastic averaging of the Schrödinger equation over these fluctuation parameters results in the qubit's decoherence, which transfers any qubit state into a statistical mixture.<sup>25,26</sup> In general, one can expect that the fluctuating environment should result in the qubit's relaxation, as well as, for instance, in the phenomenological Redfield's description of relaxation in the magnetic resonance.<sup>27</sup>

As a quantum mechanical model of the environment we consider a single-electron transistor (SET) capacitively

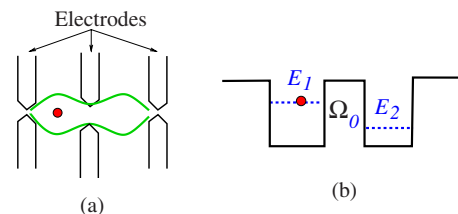


FIG. 1. (Color online) Electrostatic qubit, realized by an electron trapped in a coupled-dot system (a), and its schematic representation by a double well (b).  $\Omega_0$  denotes the coupling between the two dots.

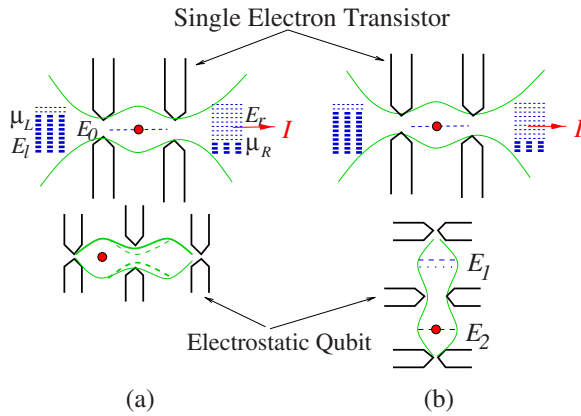


FIG. 2. (Color online) Qubit near single-electron transistor. Here  $E_{L,R}$  and  $E_0$  denote the energy levels in the left (right) reservoirs and in the quantum dot, respectively, and  $\mu_{L,R}$  are the corresponding chemical potentials. The electric current  $I$  generates fluctuations of the electrostatic opening between two dots (a), or it fluctuates the energy level of the nearest dot (b).

coupled to the qubit, e.g., Fig. 2. Such setup has been contemplated in numerous solid state quantum computing architectures where SET plays role of a readout device<sup>16,17,28,29</sup> and contains most of the generic features of a fluctuating nonequilibrium environment. The discreteness of the electron charge creates fluctuations in the electrostatic field near the SET. If the electrostatic qubit is placed near the SET, this fluctuating field should affect the qubit behavior as shown in Fig. 2. It can produce fluctuations of the tunneling coupling between the dots (off-diagonal coupling) by narrowing the electrostatic opening connecting these dots, as in Fig. 2(a), or make the energy levels of the dots fluctuate, as shown schematically in Fig. 2(b). Note that while in some regimes the SET operates as a measuring device,<sup>16,17</sup> in other regimes it corresponds purely to a source of noise. Indeed, if the energy level  $E_0$ , Fig. 2, is deeply inside the voltage bias—the case we consider in the beginning, the SET current is not modulated by the qubit electron. In this case the SET represents only the fluctuating environment affecting the qubit behavior (“pure environment”).<sup>30</sup>

A similar model of the fluctuating environment has been studied mostly for small bias (linear response) or for the environment in an equilibrium. Here, however, we consider strongly nonequilibrium case where the bias voltage applied on the SET ( $V = \mu_L - \mu_R$ ) is much larger than the level widths and the coupling between the SET and the qubit. In this limit our model can be solved exactly for both weak and strong coupling (but is still smaller than the bias voltage). This constitutes an essential advantage with regard to perturbative treatments of similar models. For instance, the results of our model can be compared in different regimes with phenomenological descriptions used in the literature. Such a comparison would allow us to determine the regions where these phenomenological models are valid.

Since our model is very simple in treatment, the decoherence and relaxation rates can be extracted from the exact solution analytically, as well as the time correlator of the electric charge inside the SET. This would make it possible

to establish a relation between the frequency-dependent fluctuation spectrum of the environment and the decoherence and the relaxation rates of the qubit, and to determine how far this relation can be extended. We expect that such a relation should not depend on a source of fluctuations. This point can be verified by a comparison with similar results obtained for equilibrium environment in the framework of the spin-boson model.<sup>1,2</sup>

It is also important to understand how the decoherence and relaxation rates depend on the frequency of the environmental fluctuations. This problem has been investigated in many phenomenological approaches for “classical” environments at equilibrium. Yet, there still exists an ambiguity in the literature related to this point for nonequilibrium environment. For instance, it was found by Levinson that the decoherence rate, generated by fluctuations of the energy level in a single quantum dot, is proportional to the spectral density of fluctuations at zero frequency.<sup>31</sup> The same result, but for a double-dot system, has been obtained by Rabenstein *et al.*<sup>32</sup> On the other hand, it follows from Redfield’s approach that the corresponding decoherence rate is proportional to the spectral density at the frequency of the qubit’s oscillations (the Rabi frequency).<sup>27</sup> Since our model is the exactly solvable one, we can resolve this ambiguity and establish the appropriate physical conditions that can result in different relations of decoherence rate to the environmental fluctuations.

The most important results of our study are related to the situation when backaction of the qubit on the environment takes place. This problem did not receive such a considerable amount of attention in the literature as, for example, the case of “inert” environment. This is in spite of a fact that the backaction always takes place in the presence of measurement. There are many questions related to the effects of a backaction. For instance, what would be a relation between decoherence (relaxation) of the qubit and the noise spectrum of the environment? Or, how decoherence is affected by a strong response of environment? We believe that our model appears to be more suitable for studying these and other problems related to the backaction than most of the other existing approaches.

The plan of this paper is as follows: Section II presents a phenomenological description of decoherence and relaxation in the framework of Bloch equations, applied to the electrostatic qubit. Section III contains a description of the model and the quantum rate-equation formalism, used for its solution. Detailed quantum mechanical derivation of these equations for a specific example is presented in Appendix A. Section IV deals with a configuration where the SET can generate only decoherence of the qubit. We consider separately the situations when SET produces fluctuations of the tunneling coupling (Rabi frequency) or of the energy levels. The results are compared with the SET fluctuation spectrum, evaluated in Appendix B. Section V deals with a configuration where the SET generates both decoherence and relaxation of the qubit. Section VI is a summary.

## II. DECOHERENCE AND RELAXATION OF A QUBIT

In this section we describe in a general phenomenological framework the effect of decoherence and relaxation on the

qubit behavior. Although the results are known, there still exists some confusion in the literature in this issue. We therefore need to define precisely these quantities and demonstrate how the corresponding decoherence and relaxation rates can be extracted from the qubit density matrix.

Let us consider an electrostatic qubit, realized by an electron trapped in coupled quantum dots, Fig. 1. This system is described by the following tunneling Hamiltonian:

$$H_{\text{qb}} = E_1 a_1^\dagger a_1 + E_2 a_2^\dagger a_2 - \Omega_0 (a_2^\dagger a_1 + a_1^\dagger a_2), \quad (1)$$

where  $a_{1,2}^\dagger, a_{1,2}$  are the creation and annihilation operators of the electron in the first or in the second dot. For simplicity we consider electrons as spinless fermions. In addition, we assume that  $a_1^\dagger a_1 + a_2^\dagger a_2 = 1$ , so that only one electron is present in the double dot. The electron wave function can be written as

$$|\Psi(t)\rangle = [b^{(1)}(t)a_1^\dagger + b^{(2)}(t)a_2^\dagger]|\mathbf{0}\rangle, \quad (2)$$

where  $b^{(1,2)}(t)$  are the probability amplitudes for finding the electron in the first or second well, obtained from the Schrödinger equation  $i\partial_t|\Psi(t)\rangle = H_{\text{qb}}|\Psi(t)\rangle$  (we adopt the units where  $\hbar=1$  and the electron charge  $e=1$ ). The corresponding density matrix,  $\sigma_{jj'}(t) = b^{(j)}(t)b^{(j')*}(t)$ , with  $j, j' = \{1, 2\}$ , is obtained from the equation  $i\partial_t \sigma = [H, \sigma]$ . This can be written explicitly as

$$\dot{\sigma}_{11} = i\Omega_0(\sigma_{21} - \sigma_{12}), \quad (3a)$$

$$\dot{\sigma}_{12} = -i\epsilon\sigma_{12} + i\Omega_0(1 - 2\sigma_{11}), \quad (3b)$$

where  $\sigma_{22}(t) = 1 - \sigma_{11}(t)$ ,  $\sigma_{21}(t) = \sigma_{12}^*(t)$ , and  $\epsilon = E_1 - E_2$ . Solving these equations one easily finds that the electron oscillates between the two dots (Rabi oscillations) with frequency  $\omega_R = \sqrt{4\Omega_0^2 + \epsilon^2}$ . For instance, for the initial conditions  $\sigma_{11}(0) = 1$  and  $\sigma_{12}(0) = 1$ , the probability of finding the electron in the second dot is  $\sigma_{22}(t) = 2(\Omega_0/\omega_R)^2(1 - \cos \omega_R t)$ . This result shows that for  $\epsilon \gg \Omega_0$  the amplitude of the Rabi oscillations is small, so the electron remains localized in its initial state.

The situation is different when the qubit interacts with the environment. In this case the (reduced) density matrix of the qubit  $\sigma(t)$  is obtained by tracing out the environment variables from the total density matrix. The question is how to modify Eqs. (3a) and (3b), written for an isolated qubit, in order to obtain the reduced density matrix of the qubit,  $\sigma(t)$ . In general one expects that the environment could affect the qubit in two different ways. First, it can destroy the off-diagonal elements of the qubit density matrix. This process is usually referred to as decoherence (or dephasing). It can be accounted for phenomenologically by introducing an additional (damping) term in Eq. (3b),

$$\dot{\sigma}_{12} = -i\epsilon\sigma_{12} + i\Omega_0(1 - 2\sigma_{11}) - \frac{\Gamma_d}{2}\sigma_{12}, \quad (4)$$

where  $\Gamma_d$  is the decoherence rate. As a result the qubit density-matrix  $\sigma(t)$  becomes a statistical mixture in the stationary limit,

$$\sigma(t) \xrightarrow{t \rightarrow \infty} \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}. \quad (5)$$

This happens for any initial conditions and even for large level displacement,  $\epsilon \gg \Omega_0, \Gamma_d$  (provided that  $\Omega_0 \neq 0$ ). Note that the statistical mixture (5) is proportional to the unity matrix and therefore it remains the same in any basis.

Secondly, the environment can put the qubit in its ground state, for instance, via photon or phonon emission. This process is usually referred to as relaxation. For a symmetric qubit we would have

$$\sigma(t) \xrightarrow{t \rightarrow \infty} \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}. \quad (6)$$

In contrast with decoherence, Eq. (5), the relaxation process puts the qubit into a pure state. That implies that the corresponding density matrix can be always written as  $\delta_{li}\delta_{lj}$  in a certain basis (the basis of the qubit eigenstates). This is, in fact, the essential difference between decoherence and relaxation. With respect to elimination of the off-diagonal density matrix elements, note that relaxation would eliminate these terms only in the qubit's eigenstate basis. In contrast, decoherence eliminates the off-diagonal density matrix element in any basis [Eq. (5)].

In fact, if the environment has some energy, it can put the qubit into an excited state. However, if the qubit is finally in a pure state, such excitation process generated by the environment affects the qubit in the same way as relaxation: it eliminates the off-diagonal density matrix elements only in a certain qubit's basis. Therefore excitation of the qubit can be described phenomenologically on the same footing as relaxation.

It is often claimed that decoherence is associated with an absence of energy transfer between the system and the environment, in contrast with relaxation (excitation). This distinction is not generally valid. For instance, if the initial qubit state corresponds to the electron in the state  $|E_2\rangle$ , Fig. 1, the final state after decoherence corresponds to an equal distribution between the two dots,  $\langle E \rangle = (E_1 + E_2)/2$ . In the case of  $E_1 \gg E_2$ , this process would require a large energy transfer between the qubit and the environment. Therefore decoherence can be consistently defined as a process leading to a statistical mixture, where all states of the system have equal probabilities [as in Eq. (5)].

The relaxation (excitation) process can be described most simply by diagonalizing the qubit Hamiltonian, Eq. (1), to obtain  $H_{\text{qb}} = E_+ a_+^\dagger a_+ + E_- a_-^\dagger a_-$ , where the operators  $a_\pm$  are obtained by the corresponding rotation of the operators  $a_{1,2}$ .<sup>30</sup> Here  $E_+$  and  $E_-$  are the ground (symmetric) and excited (antisymmetric) state energies. Then the relaxation process can be described phenomenologically in the new qubit basis  $|\pm\rangle = a_\pm^\dagger |\mathbf{0}\rangle$  as

$$\dot{\sigma}_{--}(t) = -\Gamma_r \sigma_{--}(t), \quad (7a)$$

$$\dot{\sigma}_{+-}(t) = i(E_- - E_+)\sigma_{+-}(t) - \frac{\Gamma_r}{2}\sigma_{+-}(t), \quad (7b)$$

where  $\sigma_{++}(t) = 1 - \sigma_{--}(t)$ ,  $\sigma_{-+}(t) = \sigma_{+-}^*(t)$ , and  $\Gamma_r$  is the relaxation rate.

In order to add decoherence, we return to the original qubit basis  $|1, 2\rangle = a_{1,2}^\dagger |0\rangle$  and add the damping term to the equation for the off-diagonal matrix elements, Eq. (4). We arrive at the quantum rate equation describing the qubit's behavior in the presence of both decoherence and relaxation,<sup>30,33</sup>

$$\begin{aligned} \dot{\sigma}_{11} = & i\Omega_0(\sigma_{21} - \sigma_{12}) - \Gamma_r \frac{\kappa\epsilon}{2\tilde{\epsilon}}(\sigma_{12} + \sigma_{21}) - \frac{\Gamma_r}{4} \left[ 1 + \left( \frac{\epsilon}{\tilde{\epsilon}} \right)^2 \right] \\ & \times (2\sigma_{11} - 1) + \Gamma_r \frac{\epsilon}{2\tilde{\epsilon}}, \end{aligned} \quad (8a)$$

$$\begin{aligned} \dot{\sigma}_{12} = & -i\epsilon\sigma_{12} + \left[ i\Omega_0 + \Gamma_r \frac{\kappa\epsilon}{2\tilde{\epsilon}} \right] (1 - 2\sigma_{11}) + \Gamma_r \left[ \kappa - \frac{1}{2}\sigma_{12} \right. \\ & \left. - \kappa^2(\sigma_{12} + \sigma_{21}) \right] - \frac{\Gamma_d}{2}\sigma_{12}, \end{aligned} \quad (8b)$$

where  $\tilde{\epsilon} = (\epsilon^2 + 4\Omega_0^2)^{1/2}$  and  $\kappa = \Omega_0/\tilde{\epsilon}$ . In fact, these equations can be derived in the framework of a particular model, representing an electrostatic qubit interacting with the point-contact detector and the environment, described by the Lee model Hamiltonian.<sup>33</sup>

Equations (8a) and (8b) can be rewritten in a simpler form by mapping the qubit density-matrix  $\sigma = \{\sigma_{11}, \sigma_{12}, \sigma_{21}\}$  to a “polarization” vector  $S(t)$  via  $\sigma(t) = [1 + \boldsymbol{\tau} \cdot S(t)]/2$ , where  $\tau_{x,y,z}$  are the Pauli matrices. For instance, one obtains for the symmetric case,  $\epsilon = 0$ ,

$$\dot{S}_z = -\frac{\Gamma_r}{2}S_z - 2\Omega_0 S_y, \quad (9a)$$

$$\dot{S}_y = 2\Omega_0 S_z - \frac{\Gamma_d + \Gamma_r}{2}S_y, \quad (9b)$$

$$\dot{S}_x = -\frac{\Gamma_d + 2\Gamma_r}{2}(S_x - \bar{S}_x), \quad (9c)$$

where  $\bar{S}_x = S_x(t \rightarrow \infty) = 2\Gamma_r/(\Gamma_d + 2\Gamma_r)$ . One finds that Eqs. (9a)–(9c) have a form of the Bloch equations for spin precession in the magnetic field,<sup>27</sup> where the effect of environment is accounted for by two relaxation times for the different spin components: the longitudinal  $T_1$  and the transverse  $T_2$ , related to  $\Gamma_d$  and  $2\Gamma_r$  as

$$T_1^{-1} = \frac{\Gamma_d + 2\Gamma_r}{2} \quad \text{and} \quad T_2^{-1} = \frac{\Gamma_d + \Gamma_r}{2}. \quad (10)$$

The corresponding damping rates, the so-called “depolarization” ( $\Gamma_1 = 1/T_1$ ) and the “dephasing” ( $\Gamma_2 = 1/T_2$ ), are used for phenomenological description of two-level systems.<sup>34</sup> However, neither  $\Gamma_1$  nor  $\Gamma_2$  taken alone would drive the qubit density matrix into a statistical mixture Eq. (5) or into a pure state Eq. (6).

In contrast, our definition of decoherence and relaxation (excitation) is associated with two opposite effects of the environment on the qubit: the first drives it into a statistical mixture, whereas the second drives it into a pure state. We expect therefore that such a natural distinction between decoherence and relaxation would be more useful for finding a relation between these quantities and the environmental behavior than other alternative definitions of these quantities existing in the literature.

In general, the two rates,  $\Gamma_{d,r}$ , introduced in phenomenological equations (8a), (8b), and (9a)–(9c), are consistent with our definitions of decoherence and relaxation. The only exception is the case of  $\Gamma_r = 0$  and  $\Omega_0 = 0$ , where there are no transitions between the qubit's states even in the presence of the environment (“static” qubit). One easily finds from Eqs. (3a) and (4) that  $\sigma_{12}(t) \rightarrow 0$  for  $t \rightarrow \infty$ , whereas the diagonal density-matrix elements of the qubit remain unchanged (so-called “pure dephasing”):<sup>5,34</sup>

$$\sigma(t) \xrightarrow{t \rightarrow \infty} \begin{pmatrix} \sigma_{11}(0) & 0 \\ 0 & \sigma_{22}(0) \end{pmatrix}. \quad (11)$$

Thus, if the initial probabilities of finding the qubit in each of its states are not equal,  $\sigma_{11}(0) \neq \sigma_{22}(0)$ , then the final qubit state is neither a mixture nor a pure state, but a combination of both. It implies that  $\Gamma_d$  in Eqs. (8a) and (8b) would also generate relaxation (excitation) of the qubit. Note that in this case the off-diagonal density-matrix elements, absent in Eq. (11), would reappear in a different basis. This implies that the pure dephasing<sup>5,34</sup> occurs only in a particular basis.

Let us evaluate the probability of finding the electron in the first dot,  $\sigma_{11}(t)$ . Solving Eqs. (9a)–(9c) for the initial conditions  $\sigma_{11}(0) = 1$ ,  $\sigma_{12}(0) = 0$ , we find<sup>33</sup>

$$\sigma_{11}(t) = \frac{1}{2} + \frac{e^{-\Gamma_r t/2}}{4} (C_1 e^{-e_- t} + C_2 e^{-e_+ t}), \quad (12)$$

where  $e_{\pm} = \frac{1}{4}(\Gamma_d \pm \tilde{\Omega})$ ,  $\tilde{\Omega} = \sqrt{\Gamma_d^2 - 64\Omega_0^2}$ , and  $C_{1,2} = 1 \pm (\Gamma_d/\tilde{\Omega})$ . Solving the same equations in the limit of  $t \rightarrow \infty$ , we find that the steady-state qubit density matrix is

$$\sigma(t) \xrightarrow{t \rightarrow \infty} \begin{pmatrix} 1/2 & \Gamma_r/(\Gamma_d + 2\Gamma_r) \\ \Gamma_r/(\Gamma_d + 2\Gamma_r) & 1/2 \end{pmatrix}. \quad (13)$$

Thus the off-diagonal elements of the density matrix can provide us with a ratio of relaxation to decoherence rates.<sup>33</sup>

### III. DESCRIPTION OF THE MODEL

Consider the setup shown in Fig. 2. The entire system can be described by the following tunneling Hamiltonian, represented by a sum of the qubit and SET Hamiltonians and the interaction term,  $H = H_{qb} + H_{SET} + H_{int}$ . Here  $H_{qb}$  is given by Eq. (1) and describes the qubit. The second term,  $H_{SET}$ , describes the single-electron transistor. It can be written as

$$\begin{aligned} H_{SET} = & \sum_l E_l c_l^\dagger c_l + \sum_r E_r c_r^\dagger c_r + E_0 c_0^\dagger c_0 + \sum_{l,r} (\Omega_l c_l^\dagger c_0 + \Omega_r c_r^\dagger c_0 \\ & + \text{H.c.}), \end{aligned} \quad (14)$$

where  $c_{l,r}^\dagger$  and  $c_{l,r}$  are the creation and annihilation electron



operators in the state  $E_{l,r}$  of the right or left reservoir;  $c_0^\dagger$  and  $c_0$  are those for the level  $E_0$  inside the quantum dot; and  $\Omega_{l,r}$  are the couplings between the level  $E_0$  and the level  $E_{l,r}$  in the left (right) reservoir. In order to avoid too lengthy formulas, our summation indices  $l$  and  $r$  indicate simultaneously the left and the right leads of the SET, where the corresponding summation is carried out. As follows from the Hamiltonian (14), the quantum dot of the SET contains only one level ( $E_0$ ). This assumption has been implied only for the sake of simplicity for our presentation, although our approach is well suited for a case of  $n$  levels inside the SET,  $E_0 c_0^\dagger c_0 \rightarrow \sum_n E_n c_n^\dagger c_n$ , and even when the interaction between these levels is included (providing that the latter is much less or much larger than the bias  $V$ ).<sup>35,36</sup> We also assumed a weak energy dependence of the couplings  $\Omega_{l,r} \approx \Omega_{L,R}$ .

The interaction between the qubit and the SET,  $H_{\text{int}}$ , depends on a position of the SET with respect to the qubit. If the SET is placed near the middle of the qubit, Fig. 2(a), then the tunneling coupling between two dots of the qubit in Eq. (1) decreases,  $\Omega_0 \rightarrow \Omega_0 - \delta\Omega_0$ , whenever the quantum dot of the SET is occupied by an electron. This is due to the electron's repulsive field. In this case the interaction term can be written as

$$H_{\text{int}} = \delta\Omega c_0^\dagger c_0 (a_1^\dagger a_2 + a_2^\dagger a_1). \quad (15)$$

On the other hand, in the configuration shown in Fig. 2(b) where the SET is placed near one of the dots of the qubit, the electron repulsive field displaces the qubit energy levels by  $\Delta E = U$ . The interaction terms in this case can be written as

$$H_{\text{int}} = U a_1^\dagger a_1 c_0^\dagger c_0. \quad (16)$$

Consider the initial state where all the levels in the left and the right reservoirs are filled with electrons up to the Fermi levels  $\mu_{L,R}$ , respectively. This state will be called the “vacuum” state  $|0\rangle$ . The wave function for the entire system can be written as

$$\begin{aligned} |\Psi(t)\rangle = & \left[ b^{(1)}(t) a_1^\dagger + \sum_l b_{0l}^{(1)}(t) a_1^\dagger c_0^\dagger c_l + \sum_{l,r} b_{rl}^{(1)}(t) a_1^\dagger c_r^\dagger c_l \right. \\ & + \sum_{l < l', r} b_{0rl'}^{(1)}(t) a_1^\dagger c_0^\dagger c_r^\dagger c_{l'} + \cdots + b^{(2)}(t) a_2^\dagger \\ & + \sum_l b_{0l}^{(2)}(t) a_2^\dagger c_0^\dagger c_l + \sum_{l,r} b_{rl}^{(2)}(t) a_2^\dagger c_r^\dagger c_l \\ & \left. + \sum_{l < l', r} b_{0rl'}^{(2)}(t) a_2^\dagger c_0^\dagger c_r^\dagger c_{l'} + \cdots \right] |0\rangle, \end{aligned} \quad (17)$$

where  $b^{(j)}(t)$  and  $b_\alpha^{(j)}(t)$  are the probability amplitudes to find the entire system in the state described by the corresponding creation and annihilation operators. These amplitudes are obtained from the Schrödinger equation  $i|\dot{\Psi}(t)\rangle = H|\Psi(t)\rangle$ , supplemented with the initial condition  $b^{(1)}(0) = p_1$ ,  $b^{(2)}(0) = p_2$ , and  $b_\alpha^{(j)}(0) = 0$ , where  $p_{1,2}$  are the amplitudes of the initial qubit state.

Note that Eq. (17) implies a fixed electron number ( $N$ ) in the reservoirs. At the first sight it would lead to depletion of the left reservoir of electrons over the time. Yet in the limit of  $N \rightarrow \infty$  (infinite reservoirs) the dynamics of an entire sys-

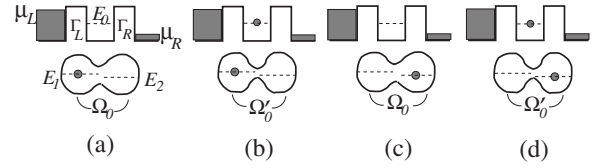


FIG. 3. The available discrete states of the entire system corresponding to the setup of Fig. 2(a).  $\Gamma_{L,R}$  denote the tunneling rates to the corresponding reservoirs and  $\Omega_0' = \Omega_0 - \delta\Omega$ .

tem reaches its steady state before such a depletion takes place.<sup>37,38</sup>

The behavior of the qubit and the SET is given by the reduced density matrix,  $\sigma_{ss'}(t)$ . It is obtained from the entire system's density matrix  $|\Psi(t)\rangle\langle\Psi(t)|$  by tracing out the (continuum) reservoir states. The space of such a reduced density matrix consists of four discrete states  $s, s' = a, b, c, d$ , shown schematically in Fig. 3 for the setup of Fig. 2(a). The corresponding density-matrix elements are directly related to the amplitudes  $b(t)$ , for instance,

$$\sigma_{aa}(t) = |b^{(1)}(t)|^2 + \sum_{l,r} |b_{lr}^{(1)}(t)|^2 + \sum_{l < l', r < r'} |b_{rr' ll'}^{(1)}(t)|^2 + \cdots, \quad (18a)$$

$$\begin{aligned} \sigma_{dd}(t) = & \sum_l |b_{0l}^{(2)}(t)|^2 + \sum_{l < l', r} |b_{0rl'}^{(2)}(t)|^2 \\ & + \sum_{l < l' < l'', r < r'} |b_{0rr' ll''}^{(2)}(t)|^2 + \cdots, \end{aligned} \quad (18b)$$

$$\begin{aligned} \sigma_{bd}(t) = & \sum_l b_{0l}^{(1)}(t) b_{0l}^{(2)*}(t) + \sum_{l < l', r} b_{0rl'}^{(1)}(t) b_{0rl'}^{(2)*}(t) \\ & + \sum_{l < l' < l'', r < r'} b_{0rr' ll''}^{(1)}(t) b_{0rr' ll''}^{(2)*}(t) + \cdots. \end{aligned} \quad (18c)$$

It was shown in Refs. 37 and 38 that the trace over the reservoir states in the system's density matrix can be performed in the large bias limit (strong nonequilibrium limit)

$$V = \mu_L - \mu_R \gg \Gamma, \Omega_0, U, \quad (19)$$

where the level (levels) of the SET carrying the current are far away from the chemical potentials, and  $\Gamma$  is the width of the level  $E_0$ . In this derivation we assumed only weak energy dependence of the transition amplitudes  $\Omega_{l,r} \equiv \Omega_{L,R}$  and the density of the reservoir states,  $\rho(E_{l,r}) = \rho_{L,R}$ . As a result we arrive at Bloch-type rate equations for the reduced density matrix without any additional assumptions. The general form of these equations is<sup>36,38</sup>

$$\begin{aligned} \dot{\sigma}_{jj'} = & i(E_{j'} - E_j) \sigma_{jj'} + i \sum_k (\sigma_{jk} \tilde{\Omega}_{k \rightarrow j'} - \tilde{\Omega}_{j \rightarrow k} \sigma_{kj'}) \\ & - \sum_{k,k'} \mathcal{P}_2 \pi \rho (\sigma_{jk} \Omega_{k \rightarrow k'} \Omega_{k' \rightarrow j'} + \sigma_{kj'} \Omega_{k \rightarrow k'} \Omega_{k' \rightarrow j}) \\ & + \sum_{k,k'} \mathcal{P}_2 \pi \rho (\Omega_{k \rightarrow j} \Omega_{k' \rightarrow j'} + \Omega_{k \rightarrow j'} \Omega_{k' \rightarrow j}) \sigma_{kk'}. \end{aligned} \quad (20)$$

Here  $\Omega_{k \rightarrow k'}$  denotes the single-electron hopping amplitude that generates the  $k \rightarrow k'$  transition. We distinguish between the amplitudes  $\tilde{\Omega}$  describing single-electron hopping between isolated states and  $\Omega$  describing transitions between isolated and continuum states. The latter can generate transitions between the isolated states of the system, but only indirectly, via two consecutive jumps of an electron, into and out of the *continuum* reservoir states (with the density of states  $\rho$ ). These transitions are represented by the third and the fourth terms of Eq. (20). The third term describes the transitions  $(k \rightarrow k' \rightarrow j)$  or  $(k \rightarrow k' \rightarrow j')$ , which cannot change the number of electrons in the collector. The fourth term describes the transitions  $(k \rightarrow j \text{ and } k' \rightarrow j')$  or  $(k \rightarrow j' \text{ and } k' \rightarrow j)$  which increase the number of electrons in the collector by 1. These two terms of Eq. (20) are analogs of the “loss” (negative) and the “gain” (positive) terms in the classical rate equations, respectively. The factor  $\mathcal{P}_2 = \pm 1$  in front of these terms is due to anticommutation of the fermions, so that  $\mathcal{P}_2 = -1$  whenever the loss or the gain terms in Eq. (20) proceed through a two-fermion state of the dot. Otherwise  $\mathcal{P}_2 = 1$ .

Note that the reduction of the time-dependent Schrödinger equation,  $i\dot{\Psi}(t) = H|\Psi(t)\rangle$ , to Eq. (20) is performed in the limit of large bias without explicit use of any Markov-type or weak coupling approximations. The accuracy of these equations is, respectively,  $\max(\Gamma, \Omega_0, U, T)/|\mu_{L,R} - E_j|$ . A detailed example of this derivation is presented in Appendix A for the case of resonant tunneling through a single level. The derivation there and in Refs. 37 and 38 were performed by assuming zero temperature in the leads,  $T=0$ . Yet, this assumption is not important in the case of large bias, providing the levels carrying the current are far away from the Fermi energies,  $|\mu_{L,R} - E_j| \gg T$ .

#### IV. NO BACKACTION ON THE ENVIRONMENT

##### A. Fluctuation of the tunneling coupling

Now we apply Eq. (20) to investigate the qubit’s behavior in the configurations shown in Fig. 2. First we consider the SET placed near the middle of the qubit, Figs. 2(a) and 3. In this case the electron current through the SET will influence the coupling between two dots of the qubit, making it fluctuate between the values  $\Omega_0$  and  $\Omega'_0 = \Omega_0 - \delta\Omega$ . The corresponding rate equations can be written straightforwardly from Eq. (20). One finds

$$\dot{\sigma}_{aa} = -\Gamma_L \sigma_{aa} + \Gamma_R \sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), \quad (21a)$$

$$\dot{\sigma}_{bb} = -\Gamma_R \sigma_{bb} + \Gamma_L \sigma_{aa} - i\Omega'_0(\sigma_{bd} - \sigma_{db}), \quad (21b)$$

$$\dot{\sigma}_{cc} = -\Gamma_L \sigma_{cc} + \Gamma_R \sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), \quad (21c)$$

$$\dot{\sigma}_{dd} = -\Gamma_R \sigma_{dd} + \Gamma_L \sigma_{cc} - i\Omega'_0(\sigma_{db} - \sigma_{bd}), \quad (21d)$$

$$\dot{\sigma}_{ac} = -i\epsilon_0 \sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \Gamma_L \sigma_{ac} + \Gamma_R \sigma_{bd}, \quad (21e)$$

$$\dot{\sigma}_{bd} = -i\epsilon_0 \sigma_{bd} - i\Omega'_0(\sigma_{bb} - \sigma_{dd}) - \Gamma_R \sigma_{bd} + \Gamma_L \sigma_{ac}, \quad (21f)$$

where  $\Gamma_{L,R} = 2\pi|\Omega_{L,R}|^2 \rho_{L,R}$  are the tunneling rates from the reservoirs and  $\epsilon_0 = E_1 - E_2$ .

These equations display explicitly the time evolution of the SET and the qubit. The evolution of the former is driven by the first two terms in Eqs. (21a)–(21d). They generate charge fluctuations inside the quantum dot of the SET (the transitions  $a \leftrightarrow b$  and  $c \leftrightarrow d$ ), described by the classical Boltzmann-type dynamics. The qubit’s evolution is described by the Bloch-type terms [cf. Eqs. (3a) and (3b)], generating the qubit transitions ( $a \leftrightarrow c$  and  $b \leftrightarrow d$ ). Thus Eqs. (21a)–(21f) are quite general, since they described fluctuations of the tunneling coupling driven by the Boltzmann-type dynamics.

The resulting time evolution of the qubit is given by the qubit (reduced) density matrix as follows

$$\sigma_{11}(t) = \sigma_{aa}(t) + \sigma_{bb}(t), \quad (22a)$$

$$\sigma_{12}(t) = \sigma_{ac}(t) + \sigma_{bd}(t), \quad (22b)$$

and  $\sigma_{22}(t) = 1 - \sigma_{11}(t)$ .

Similarly, the charge fluctuations of SET are determined by the probability of finding the SET occupied,

$$P_1(t) = \sigma_{bb}(t) + \sigma_{dd}(t). \quad (23)$$

It is given by the equation

$$\dot{P}_1(t) = \Gamma_L - \Gamma P_1(t), \quad (24)$$

obtained straightforwardly from Eqs. (21a)–(21f). Here  $\Gamma = \Gamma_L + \Gamma_R$  is the total width. The same equation for  $P_1(t)$  can be obtained if the qubit is decoupled from the SET ( $\delta\Omega = 0$ ). Thus there is no backaction of the qubit on the charge fluctuations inside the SET in the limit of large bias voltage.

Consider first the stationary limit,  $t \rightarrow \infty$ , where  $\dot{P}_1(t) \rightarrow 0$  and  $\dot{\sigma}(t) \rightarrow 0$ . It follows from Eq. (24) that the probability of finding the SET occupied in this limit is  $\bar{P}_1 = \Gamma_L / \Gamma$ . This implies that the fluctuations of the coupling  $\Omega_0$ , induced by the SET, would take place around the average value  $\Omega = \Omega_0 - \bar{P}_1 \delta\Omega$ .

With respect to the qubit in the stationary limit, one easily obtains from Eqs. (21a)–(21f) that the qubit density matrix always becomes the statistical mixture (5), when  $t \rightarrow \infty$ . This takes place for any initial conditions and any values of the qubit and the SET parameters. Therefore the effect of the fluctuating charge inside the SET does not lead to relaxation of the qubit, but rather to its decoherence.

It is important to note, however, that for the aligned qubit,  $\epsilon=0$ , the decoherence due to fluctuations of the tunneling coupling  $\Omega_0$  is not complete. Indeed, it follows from Eqs. (21a)–(21f) that  $d/dt[\text{Re } \sigma_{12}(t)] = 0$ . The reason is that the corresponding operator,  $a_1^\dagger a_2 + a_2^\dagger a_1$ , commutes with the total Hamiltonian,  $H = H_{\text{qb}} + H_{\text{SET}} + H_{\text{int}}$ , Eqs. (1), (14), and (15), for  $E_1 = E_2$ . As a result,  $\text{Re } \sigma_{12}(t) = \text{Re } \sigma_{12}(0)$ .

In order to determine the decoherence rate analytically, we perform a Laplace transform on the density matrix,  $\bar{\sigma}(E) = \int_0^\infty \sigma(t) \exp(-iEt) dt$ . Then solving Eqs. (21a)–(21f) we

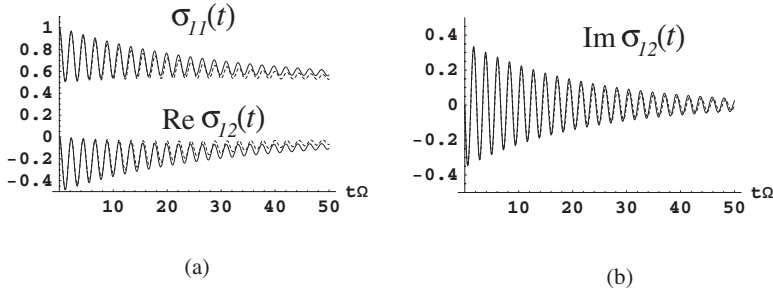


FIG. 4. The occupation probability of the first dot of the qubit for  $\epsilon=2\Omega$ ,  $\Gamma_L=\Omega$ ,  $\Gamma_R=2\Omega$ , and  $\delta\Omega=0.5\Omega$ . The solid line is the exact result, whereas the dashed line is obtained from the Bloch-type rate equations with the decoherence rate given by Eq. (30).

can determine the decoherence rate from the locations of the poles of  $\tilde{\sigma}(E)$  in the complex  $E$  plane. Consider, for instance, the case of  $\epsilon_0=0$  and the symmetric SET,  $\Gamma_L=\Gamma_R=\Gamma/2$ . One finds from Eqs. (21a)–(21f) and (22a) that

$$\tilde{\sigma}_{11}(E) = \frac{i}{2E} + \frac{i(E-2\Omega+i\Gamma)}{4(E-2\Omega+i\Gamma/2)^2 + \Gamma^2 - (2\delta\Omega)^2} + \frac{i(E+2\Omega+i\Gamma)}{4(E+2\Omega+i\Gamma/2)^2 + \Gamma^2 - (2\delta\Omega)^2}. \quad (25)$$

Upon performing the inverse Laplace transform,

$$\sigma_{11}(t) = \int_{-\infty+i0}^{\infty+i0} \tilde{\sigma}_{11}(E) e^{-iEt} \frac{dE}{2\pi i}, \quad (26)$$

and closing the integration contour around the poles of the integrand, we obtain for  $\Gamma > 2\delta\Omega$  and  $t \gg 1/\Gamma$

$$\sigma_{11}(t) - (1/2) \propto e^{-(\Gamma - \sqrt{\Gamma^2 - 4\delta\Omega^2})t/2} \sin(2\Omega t). \quad (27)$$

Comparing this result with Eq. (12) we find that the decoherence rate is

$$\Gamma_d = 2(\Gamma - \sqrt{\Gamma^2 - 4\delta\Omega^2}) \xrightarrow{\Gamma \gg \delta\Omega} (2\delta\Omega)^2/\Gamma. \quad (28)$$

For  $\epsilon_0 \neq 0$  and  $\epsilon_0, \Gamma \ll \Omega$  the decoherence rate  $\Gamma_d$  is multiplied by an additional factor  $[1 - (\epsilon_0/2\Omega)^2]$ .

In a general case,  $\Gamma_L \neq \Gamma_R$ , we obtain in the same limit ( $\Gamma_{L,R} \gg \delta\Omega$ ) for the decoherence rate

$$\Gamma_d = \frac{(4\delta\Omega)^2}{1 + \left(\frac{\epsilon_0}{2\Omega}\right)^2} \frac{\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^3}. \quad (29)$$

It is interesting to compare this result with the fluctuation spectrum of the charge inside the SET, Eq. (B8), Appendix B. We find

$$\Gamma_d = 2(\delta\omega_R)^2 S_Q(0), \quad (30)$$

where  $\omega_R = \sqrt{4\Omega^2 + \epsilon_0^2}$  is the Rabi frequency. The latter represents the energy splitting in the diagonalized qubit Hamiltonian. Thus  $\delta\omega_R$  corresponds to the amplitude of energy level fluctuations in a single dot.

Although Eq. (30) has been obtained for small fluctuations  $\delta\omega_R$ , it might be approximately correct even if  $\delta\omega_R$  is of the order of  $\Gamma$ . It is demonstrated in Fig. 4, where we compare  $\sigma_{11}(t)$  and  $\sigma_{12}(t)$ , obtained from Eqs. (21a)–(21f), (22a), and (22b) (solid line) with those from Eqs. (3a) and (4) (dashed line) for the decoherence rate  $\Gamma_d$  given by Eq.

(30). The initial conditions correspond to  $\sigma_{11}(0)=1$  and  $\sigma_{12}(0)=0$  [respectively,  $\sigma_{aa}(0)=\Gamma_R/\Gamma$  and  $\sigma_{bb}(0)=\Gamma_R/\Gamma$ ].

In the case of aligned qubit, however,  $\text{Re } \sigma_{12}(t) = \text{Re } \sigma_{12}(0)$ , as was explained above. On the other hand, one always obtains from Eqs. (3a) and (4) that  $\text{Re}[\sigma_{12}(t \rightarrow \infty)] = 0$ . Therefore the phenomenological Bloch equations are not applicable for evaluation of  $\text{Re}[\sigma_{12}(t)]$ , even in the weak-coupling limit (besides the case of  $\text{Re}[\sigma_{12}(t=0)]=0$ ).

In the large coupling regime ( $\delta\Omega \gg \Gamma$ ) the phenomenological Bloch equations, Eqs. (3a) and (4), cannot be used, as well. Consider for simplicity the case of  $\epsilon=0$  and  $\Gamma_{L,R}=\Gamma/2$ . Then one finds from Eq. (27) that the damping oscillations between the two dots take place at two different frequencies,  $2\Omega \pm \sqrt{(\delta\Omega)^2 - (\Gamma/2)^2}$ , instead of the one frequency,  $\omega_R=2\Omega$ , given the Bloch equations. Moreover, Eq. (30) does not reproduce the decoherence (damping) rate in this limit. Indeed, one obtains from Eq. (28) that the decoherence rate  $\Gamma_d=2\Gamma$  for  $\delta\Omega > \Gamma/2$ , so  $\Gamma_d$  does not depend on the coupling ( $\delta\Omega$ ) at all.

## B. Fluctuation of the energy level

Consider the SET placed near one of the qubit dots, as shown in Fig. 2(b). In this case the qubit-SET interaction term is given by Eq. (16). As a result the energy level  $E_1$  will fluctuate under the influence of the fluctuations of the electron charge inside the SET. The available discrete states of the entire system are shown in Fig. 5. Using Eq. (20) we can write the rate equations, similar to Eqs. (21a)–(21f),

$$\dot{\sigma}_{aa} = -\Gamma'_L \sigma_{aa} + \Gamma'_R \sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), \quad (31a)$$

$$\dot{\sigma}_{bb} = -\Gamma'_R \sigma_{bb} + \Gamma'_L \sigma_{aa} - i\Omega_0(\sigma_{bd} - \sigma_{db}), \quad (31b)$$

$$\dot{\sigma}_{cc} = -\Gamma_L \sigma_{cc} + \Gamma_R \sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), \quad (31c)$$

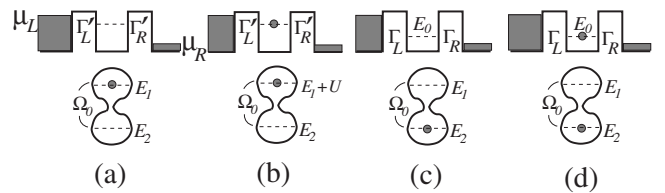


FIG. 5. The available discrete states of the entire system for the configuration shown in Fig. 2(b). Here  $U$  is the repulsion energy between the electrons.

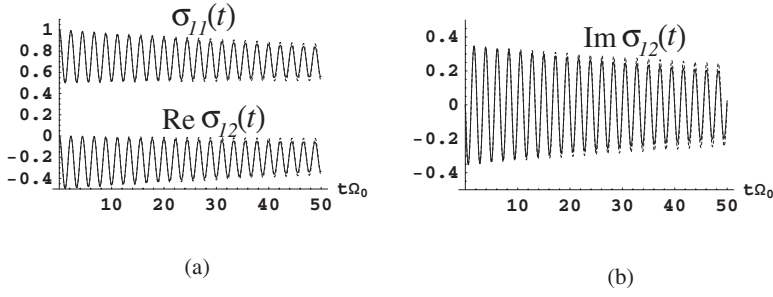


FIG. 6. The probability of finding the electron in the first dot of the qubit for  $\epsilon=2\Omega_0$ ,  $\Gamma_L=\Omega_0$ ,  $\Gamma_R=2\Omega_0$ , and  $U=0.5\Omega_0$ . The solid line is the exact result, whereas the dashed line is obtained from the Bloch-type rate equations with the decoherence rate given by Eq. (33).

$$\dot{\sigma}_{dd} = -\Gamma_R \sigma_{dd} + \Gamma_L \sigma_{cc} - i\Omega_0(\sigma_{db} - \sigma_{bd}), \quad (31d)$$

$$\dot{\sigma}_{ac} = -i\epsilon_0 \sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \frac{\Gamma_L + \Gamma'_L}{2} \sigma_{ac} + \sqrt{\Gamma_R \Gamma'_R} \sigma_{bd}, \quad (31e)$$

$$\begin{aligned} \dot{\sigma}_{bd} = & -i(\epsilon_0 + U) \sigma_{bd} - i\Omega_0(\sigma_{bb} - \sigma_{dd}) - \frac{\Gamma_R + \Gamma'_R}{2} \sigma_{bd} \\ & + \sqrt{\Gamma_L \Gamma'_L} \sigma_{ac}, \end{aligned} \quad (31f)$$

where  $\Gamma'_{L,R}$  is the tunneling rate at the energy  $E_0 + U$ .<sup>39</sup>

Let us assume that  $\Gamma'_{L,R} = \Gamma_{L,R}$ . Then it follows from Eqs. (31a)–(31f) that the behavior of the charge inside the SET is not affected by the qubit, the same as in the previous case of the Rabi frequency fluctuations. Also the qubit density matrix becomes the mixture (5) in the stationary state for any values of the qubit and the SET parameters. Hence, there is no qubit relaxation in this case either [except for the static qubit,  $\Omega_0=0$ , and  $\sigma_{11}(0) \neq \sigma_{22}(0)$ , Eq. (11)].

Since according to Eq. (24), the probability of finding an electron inside the SET in the stationary state is  $\bar{P}_1 = \Gamma_L / \Gamma$ , the energy level  $E_1$  of the qubit is shifted by  $\bar{P}_1 U$ . Therefore it is useful to define the “renormalized” level displacement,  $\epsilon = \epsilon_0 + \bar{P}_1 U$ .

As in the previous case we use the Laplace transform,  $\sigma(t) \rightarrow \tilde{\sigma}(E)$ , in order to determine the decoherence rate analytically. In the case of  $\Gamma_L = \Gamma_R = \Gamma/2$  and  $\epsilon=0$  we obtain from Eqs. (31a)–(31f)

$$\tilde{\sigma}_{11}(E) = \frac{i}{2E} + \frac{i}{2E + \frac{32(E+i\Gamma)\Omega_0^2}{U^2 - 4E(E+i\Gamma)}}. \quad (32)$$

The position of the pole in the second term of this expression determines the decoherence rate. In contrast with Eq. (25), however, the exact analytical expression for the decoherence rate ( $\Gamma_d$ ) is complicated, since it is given by a cubic equation. We therefore evaluate  $\Gamma_d$  in a different way, by substituting  $E = \pm 2\Omega_0 - i\gamma$  in the second term of Eq. (32) and then expanding the latter in powers of  $\gamma$  by keeping only the first two terms of this expansion. The decoherence rate  $\Gamma_d$  is related to  $\gamma$  by  $\Gamma_d = 4\gamma$ , as follows from Eq. (12). Then we obtain

$$\Gamma_d = \begin{cases} \frac{U^2 \Gamma}{2(\Gamma^2 + 4\Omega_0^2)} & \text{for } U \ll (\Omega_0^2 + \Gamma\Omega_0)^{1/2} \\ \frac{64\Gamma\Omega_0^2}{U^2 + 16\Omega_0^2} & \text{for } U \gg (\Omega_0^2 + \Gamma\Omega_0)^{1/2}. \end{cases} \quad (33)$$

In general, if  $\Gamma_L \neq \Gamma_R$ , one finds from Eqs. (31a)–(31f) that  $\Gamma_d = 2U^2 \Gamma_L \Gamma_R / [\Gamma(\Gamma^2 + 4\Omega_0^2)]$  for  $U \ll (\Omega_0^2 + \Gamma\Omega_0)^{1/2}$ . The same as in the previous case, Eq. (30), the decoherence rate in a weak-coupling limit is related to the fluctuation spectrum of the SET,  $S_Q(\omega)$ , Eq. (B8), but now taken at a different frequency,  $\omega = 2\Omega_0$ . The latter corresponds to the level splitting of the diagonalized qubit’s Hamiltonian,  $\omega_R$ . Thus,

$$\Gamma_d = U^2 S_Q(\omega_R), \quad (34)$$

which can be applied also for  $\epsilon \neq 0$ . This is illustrated by Fig. 6 which shows  $\sigma_{11}(t)$  obtained from Eqs. (31a)–(31f), (22a), and (22b) (solid line) with Eqs. (3a) and (4) (dashed line) for the decoherence rate  $\Gamma_d$  given by Eq. (34). As in the previous case, shown in Fig. 4, the initial conditions correspond to  $\sigma_{11}(0)=1$  and  $\sigma_{12}(0)=0$  [respectively,  $\sigma_{aa}(0)=\Gamma_R/\Gamma$  and  $\sigma_{bb}(0)=\Gamma_R/\Gamma$ ]. One finds from Fig. 6 that Eq. (34) can be used for an estimation of  $\Gamma_d$  even for  $U \sim \Gamma, \Omega_0$ .

In contrast with the tunneling-coupling fluctuations, Eq. (30), where the decoherence rate is given by  $S_Q(0)$ , the fluctuations of the qubit’s energy level generate the decoherence rate, determined by the fluctuation spectrum at Rabi frequency,  $S_Q(\omega_R)$ , Eq. (34). A similar distinction between the decoherence rates generated by different components of the fluctuating field exists in a phenomenological description of magnetic resonance.<sup>27</sup> One can understand this distinction by diagonalizing the qubit’s Hamiltonian. In this case the Rabi frequency,  $\omega_R$ , becomes the level splitting of the qubit’s states  $|\pm\rangle = (|1\rangle \pm |2\rangle) / \sqrt{2}$  (for  $\epsilon=0$ ). So in this basis, the tunneling-coupling fluctuations correspond to simultaneous fluctuations of the energy levels in both dots. Since these fluctuations are “in phase,” we could expect that the corresponding dephasing rate is determined by spectral density at zero frequency. In fact, it looks like as fluctuations of a single dot state, considered by Levinson in a weak-coupling limit.<sup>31</sup> On the other hand by fluctuating the energy level in one of the dots only, one can anticipate that the corresponding dephasing rate is determined by the fluctuation spectrum at the Rabi frequency,  $\omega_R$ , Eq. (34), which is a frequency of the interdot transitions.

Since  $\omega_R$  can be controlled by the qubit’s level displacement,  $\epsilon$ , the relation (34) can be implied by using qubit for a



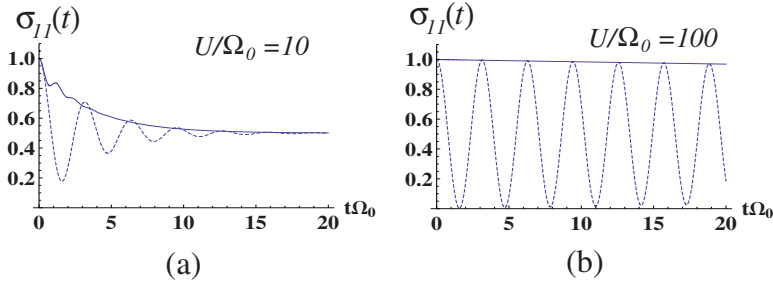


FIG. 7. (Color online) The probability of finding the electron in the first dot of the qubit for  $\epsilon=0$ ,  $\Gamma_L=\Gamma_R=\Omega_0$ , and  $U$ , as given by Eqs. (31a)–(31f) (solid line) and from the Bloch-type equations (dashed line) with the decoherence rate given by Eq. (33).

measurement of the shot-noise spectrum of the environment.<sup>18,19,40</sup> For instance, it can be done by attaching a qubit to reservoirs at different chemical potentials. The corresponding resonant current, which would flow through the qubit in this case, can be evaluated via a simple analytical expression<sup>13</sup> that includes explicitly the decoherence rate, Eq. (34). Thus by measuring this current for different level displacement of the qubit ( $\epsilon_0$ ), one can extract the spectral density of the fluctuating environment acting on the qubit.<sup>18</sup>

Although Eq. (34) for the decoherence rate has been obtained by using a particular mechanism for fluctuations of the qubit's energy levels, we suggest that this mechanism is quite general. Indeed, the rate equations (31a)–(31f) can describe any fluctuating media near a qubit, driven by the Boltzmann type of equations. Therefore it is rather natural to assume that Eq. (34) would be valid for any type of such (classical) environment in the weak-coupling limit. This implies that the decoherence rate is always determined via the spectral density of a fluctuating qubit's level, whereas the nature of a particular medium inducing these fluctuations would be irrelevant. In order to substantiate this point it is important to compare Eq. (34) with the corresponding decoherence rate induced by the thermal environment in the framework of the spin-boson model. In a weak damping limit this model predicts<sup>1,2</sup>  $T_1^{-1}=T_2^{-1}=(q_0^2/2)S(\omega_R)$ , where  $q_0$  is a coupling of the medium with the qubit levels ( $q_0$  corresponds to  $U$  in our case) and  $S(\omega)$  is a spectral density. Using Eq. (10) one finds that this result coincides with Eq. (34).

### C. Strong-coupling limit and localization

Let us consider the limit of  $U \gg (\Omega_0^2 + \Gamma\Omega_0)^{1/2}$ . Our rate equations (31a)–(31f) are perfectly valid in this region, providing only that  $E_0 + U$  is deeply inside of the potential bias, Eq. (19). We find from Eq. (33) that the decoherence rate is not directly related to the spectrum of fluctuations in the strong-coupling limit. In addition, the effective frequency of the qubit's Rabi oscillations ( $\omega_R^{eff}$ ) decreases in this limit. Indeed, by using Eqs. (32) and (26), one finds that the main contribution to  $\sigma_{11}(t)$  is coming from a pole of  $\tilde{\sigma}_{11}(E)$ , which lies on the imaginary axis. This implies that the effective frequency of Rabi oscillations strongly decreases when  $U \gg (\Omega_0^2 + \Gamma\Omega_0)^{1/2}$ . In addition, the decoherence rate  $\Gamma_d \rightarrow 0$  in the same limit, Eq. (33). As a result, the electron would localize in the initial qubit state, Fig. 7.

The results displayed in this figure show that the solution of the Bloch-type rate equations, with the decoherence rate given by Eq. (33), represents damped oscillations (dashed

line). It is very far from the exact result (solid line), obtained from Eqs. (31a)–(31f) and corresponding to the electron localization in the first dot. The latter is a result of an effective decrease of the Rabi frequency for large  $U$  that slows down electron transitions between the dots. Thus such an environment-induced localization is different from the Zeno-type effect (unlike an assumption of Ref. 12). Indeed, the Zeno effect takes place whenever the decoherence rate is much larger than the coupling between the qubit's states.<sup>13,33</sup> However, the decoherence rate in the strong-coupling limit is much smaller than the coupling  $\Omega_0$ . In fact, the localization shown in Fig. 7 is rather similar to that in the spin-boson model.<sup>1,2</sup> It shows that in spite of their differences, both models trace the same physics of the backaction of the environment (SET) on the qubit.

## V. BACKACTION OF THE QUBIT ON THE ENVIRONMENT

### A. Weak backaction effect

Now we investigate a weak dependence of the width's  $\Gamma_{L,R}$  on the energy  $U$ , Fig. 5. We keep only the linear term,  $\Gamma'_{L,R} = \Gamma_{L,R} + \alpha_{L,R}U$ , by assuming that  $U$  is small. (A similar model has been considered in Refs. 28 and 41.) In contrast with the previous examples, where the widths have not been dependent on the energy, the qubit's oscillation would affect the SET current and its charge correlator. A more interesting case corresponds to  $\alpha_L \neq \alpha_R$ . Let us take for simplicity  $\alpha_L = 0$  and  $\alpha_R = \alpha \neq 0$ .

Similarly to the previous case we introduce the renormalized level displacement,  $\epsilon = \epsilon_0 - (\Gamma_L/\Gamma)U$ , where  $\epsilon=0$  corresponds to the aligned qubit. Solving Eqs. (31a)–(31f) in the steady-state limit,  $\bar{\sigma} = \sigma(t \rightarrow \infty)$ , and keeping only the first term in expansion in powers of  $U$ , we find for the reduced density matrix of the qubit, Eqs. (22a) and (22b), the following:

$$\bar{\sigma} = \begin{pmatrix} \frac{1}{2} - \frac{\alpha\epsilon}{4\Gamma_R} & \frac{\alpha\Omega_0(1+c\alpha U)}{2\Gamma_R} \\ \frac{\alpha\Omega_0(1+c\alpha U)}{2\Gamma_R} & \frac{1}{2} + \frac{\alpha\epsilon}{4\Gamma_R} \end{pmatrix}, \quad (35)$$

where  $c = (\alpha\epsilon - 2\Gamma)/(4\Gamma_R\Gamma)$ . It follows from Eq. (35) that the qubit's density matrix in the steady state is no longer a mixture, Eq. (5). Indeed, the probability to occupy the lowest level is always larger than 1/2 and  $\bar{\sigma}_{12} \neq 0$ . This implies that relaxation takes place together with decoherence. The ratio

of the relaxation and decoherence rates is given by the off-diagonal terms of the reduced density matrix of the qubit. For  $\epsilon=0$  one finds from Eq. (13) that  $\Gamma_d/\Gamma_r = \bar{\sigma}_{12}^{-1} - 2$ .

In order to find a relation between the decoherence and relaxation rates,  $\Gamma_{d,r}$ , and the fluctuation spectrum of the qubit energy level,  $S_Q(\omega)$ , we first evaluate the total damping rate of the qubit's oscillations ( $\gamma$ ). Using Eq. (12) we find that this quantity is related to the decoherence and relaxation rates by  $\gamma = (\Gamma_d + 2\Gamma_r)/4$ . The same as in the previous case the rate  $\gamma$  is determined by poles of Laplace transformed density matrix  $\sigma(t) \rightarrow \bar{\sigma}(E)$  in the complex  $E$  plane. Consider for simplicity the case of  $\epsilon=0$  and  $\Gamma_L = \Gamma_R = \Gamma/2$ . Performing the Laplace transform of Eqs. (31a)–(31f) we look for the poles of  $\sigma_{11}(E)$  at  $E = \pm 2\Omega_0 - i\gamma$  by assuming that  $\gamma$  is small. We obtain

$$\Gamma_d + 2\Gamma_r = \frac{U^2}{2(\Gamma^2 + 4\Omega_0^2)} \left[ \Gamma - \alpha U \frac{\Gamma^2 - 4\Omega_0^2}{2(\Gamma^2 + 4\Omega_0^2)} \right] \quad (36)$$

for  $U \ll \Omega_0$ .

Now we evaluate the correlator of the charge inside the SET,  $S_Q(\omega)$  which induces the energy-level fluctuations of the qubit. Using Eqs. (31a)–(31f) and (B6) we find

$$S_Q(\omega) = \frac{\Gamma}{2(\Gamma^2 + \omega^2)} - \alpha U \frac{\Gamma^2 - \omega^2}{4(\Gamma^2 + \omega^2)^2} \quad (37)$$

for  $\alpha U \ll \Gamma$ . Therefore in the limit of  $U \ll \Omega_0$  and  $\alpha U \ll \Gamma$  the total damping rate of the qubit's oscillations is directly related to the spectral density of the fluctuations spectrum taken at the Rabi frequency,

$$\Gamma_d + 2\Gamma_r = U^2 S_Q(2\Omega_0). \quad (38)$$

This represents a generalization of Eq. (34) for the case of a weak backaction of qubit oscillations on the spectral density of the environment. As a result, the qubit displays relaxation together with decoherence. It is remarkable that the total qubit's damping rate is still given by the fluctuation spectrum of the SET (environment) modulated by the qubit. Note that Eq. (38) can be applied only if the modulation of the tunneling rate through the SET (tunneling current) is small  $\alpha U \ll \Gamma$ , in addition to a weak distortion of the qubit ( $U \ll \Omega_0$ ).

In the case of strong backaction of the qubit on the environment the decoherence and relaxation rates of the qubit are not directly related to the fluctuation spectrum of the environment, even if the distortion of the qubit is small. This point is illustrated by the following example.

### B. Strong backaction

Until now we considered the case where  $E_0 + U \ll \mu_L$ , so that the interacting electron of the SET remains deeply inside the voltage bias. If, however, the interaction  $U$  between the qubit and the SET is such that  $E_0 + U \gg \mu_L$ , the qubit's oscillation would strongly affect the fluctuation of charge inside the SET. Indeed, the current through the SET is blocked whenever the level  $E_1$  of the qubit is occupied, Fig. 8. In fact, this case can be treated with small modification of the rate equations (31a)–(31f), if only  $\mu_L - E_0 \gg \Gamma$  and  $E_0 + U - \mu_L \gg \Gamma$ , where  $E_0$  is a level of the SET carrying the current.

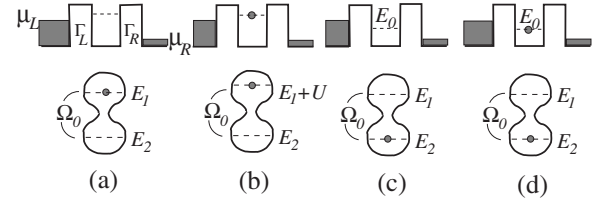


FIG. 8. The available discrete states of the entire system when the electron-electron repulsive interaction  $U$  breaks off the current through the SET.

The corresponding quantum rate equations describing the system are obtained directly from Eq. (20). Assuming that the widths  $\Gamma_{L,R}$  are energy independent we find<sup>16</sup>

$$\dot{\sigma}_{aa} = (\Gamma_L + \Gamma_R)\sigma_{bb} - i\Omega_0(\sigma_{ac} - \sigma_{ca}), \quad (39a)$$

$$\dot{\sigma}_{bb} = -(\Gamma_R + \Gamma_L)\sigma_{bb} - i\Omega_0(\sigma_{bd} - \sigma_{db}), \quad (39b)$$

$$\dot{\sigma}_{cc} = -\Gamma_L\sigma_{cc} + \Gamma_R\sigma_{dd} - i\Omega_0(\sigma_{ca} - \sigma_{ac}), \quad (39c)$$

$$\dot{\sigma}_{dd} = -\Gamma_R\sigma_{dd} + \Gamma_L\sigma_{cc} - i\Omega_0(\sigma_{db} - \sigma_{bd}), \quad (39d)$$

$$\dot{\sigma}_{ac} = -i\epsilon_0\sigma_{ac} - i\Omega_0(\sigma_{aa} - \sigma_{cc}) - \frac{\Gamma_L}{2}\sigma_{ac} + \Gamma_R\sigma_{bd}, \quad (39e)$$

$$\dot{\sigma}_{bd} = -i(\epsilon_0 + U)\sigma_{bd} - i\Omega_0(\sigma_{bb} - \sigma_{dd}) - \left(\Gamma_R + \frac{\Gamma_L}{2}\right)\sigma_{bd}. \quad (39f)$$

Solving Eqs. (39a)–(39f) in the stationary limit,  $\bar{\sigma} = \sigma(t \rightarrow \infty)$ , and introducing the renormalized level displacement,  $\epsilon = \epsilon_0 - U\Gamma_L/(2\Gamma)$ , we obtain for the qubit's density matrix, Eqs. (22a) and (22b), in the steady state as follows:

$$\bar{\sigma}_{11} = \frac{1}{2} - \frac{8\epsilon U}{16\epsilon^2 + 8U\epsilon + 48\Omega_0^2 + 9(U^2 + \Gamma^2)}, \quad (40a)$$

$$\bar{\sigma}_{12} = \frac{12U\Omega_0}{16\epsilon^2 + 8U\epsilon + 48\Omega_0^2 + 9(U^2 + \Gamma^2)}, \quad (40b)$$

where for simplicity we considered the symmetric case,  $\Gamma_L = \Gamma_R = \Gamma/2$ . It follows from Eqs. (40a) and (40b) that similarly to the previous example, the qubit's density matrix is no longer a mixture (5). The relaxation takes place together with decoherence in this case too.

Let us consider weak distortion of the qubit by the SET,  $U < \Omega_0$ . Although the values of  $U$  are restricted from below ( $U \gg \Gamma + \mu_L - E_0$ ), this limit can be achieved if the level  $E_0$  is close to the Fermi energy, providing only that  $\mu_L - E_0 \gg \Gamma$  and  $\Gamma \ll U$ . Now we evaluate  $\sigma_{11}(t)$  with the rate equations (39a)–(39f) and then compare it with the same quantity obtained from the Bloch equations, Eq. (12), where  $\Gamma_{d,r}$  are given by Eqs. (34) and (13). The corresponding charge correlator,  $S_Q(\omega_R)$ , is evaluated by Eqs. (B6) and (39a)–(39f). As an example, we take symmetric qubit with aligned levels,

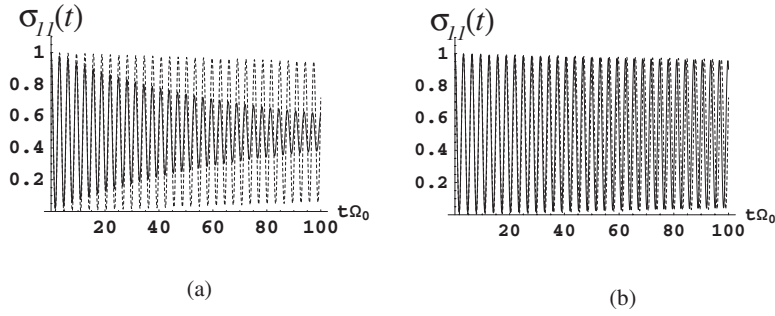


FIG. 9. (a) The probability of finding the electron in the first dot of the qubit for  $\epsilon=0$ ,  $\Gamma_L=\Gamma_R=0.05\Omega_0$ , and  $U=0.5\Omega_0$ . The solid line is obtained from Eqs. (39a)–(39f), whereas the dashed line corresponds to Eq. (12) with  $\Gamma_d$  given by Eq. (34); (b) the same for the case, shown in Fig. 5, where the solid line corresponds to Eqs. (31a)–(31f).

$\epsilon=0$ ,  $\Gamma_L=\Gamma_R=0.05\Omega_0$ , and  $U=0.5\Omega_0$ . The decoherence and relaxation rates, corresponding to these parameters, are, respectively,  $\Gamma_d/\Omega_0=0.0038$  and  $\Gamma_r/\Omega_0=0.00059$ .

The results are presented in Fig. 9(a). The solid line shows  $\sigma_{11}(t)$ , obtained from the rate equations (39a)–(39f), where the dashed line is the same quantity obtained from Eq. (12). We find that Eq. (34) [or (38)] underestimates the actual damping rate of  $\sigma_{11}(t)$  by an order of magnitude. This lies in a sharp contrast with the previous case, where the energy level of the SET is not distorted by the qubit,  $\Gamma'_{L,R}=\Gamma_{L,R}$ , Fig. 5. Indeed, in this case  $\sigma_{11}(t)$  obtained Eq. (12) with  $\Gamma_d$  given by Eq. (34) and  $\Gamma_r=0$  agrees very well with that obtained from the rate equations (31a)–(31f) as shown in Fig. 9(b).

Such an example clearly illustrates that the decoherence is not related to the fluctuation spectrum of the environment, whenever the environment is strongly affected by the qubit, even if the coupling with a qubit is small. This is a typical case of measurement, corresponding to a noticeable response of the environment to the qubit's state (a “signal”).

## VI. SUMMARY

In this paper we propose a simple model describing a qubit interacting with fluctuating environment. The latter is represented by a single-electron transistor (SET) in close proximity of the qubit. Then the fluctuations of the charge inside the SET generate fluctuating field acting on the qubit. In the limit of large bias voltage, the Schrödinger equation for the entire system is reduced to the Bloch-type rate equations. The resulting equations are very simple, so that one can easily analyze the limits of weak and strong coupling of the qubit with the SET.

We considered separately two different cases: (a) there is no backaction of the qubit on the SET behavior, so that the latter represents a “pure environment;” and (b) the SET behavior depends on the qubit's state. In the latter case the SET can “measure” the qubit. The setup corresponding to the pure environment is realized when the energy level of the SET carrying the current lies deeply inside the potential bias. The second (measurement) regime of the SET is realized when the tunneling widths of the SET are energy dependent, or when the energy level of the SET carrying the current is close enough to the Fermi level of the corresponding reservoir. Then the electron-electron interaction between the qubit and the SET modulates the electron current through the SET.

In the case of the pure environment (“no-measurement” regime) we investigate separately two different configura-

tions of the qubit with respect to the SET. In the first one the SET produces fluctuations of the off-diagonal coupling (Rabi frequency) between two qubit's states. In the second configuration the SET produces fluctuations of the qubit's energy levels. In both cases we find no relaxation of the qubit, despite the energy transfer between the qubit and the SET can take place. As a result the qubit always turns asymptotically to the statistical mixture. We also found that in both cases the decoherence rate of the qubit in the weak-coupling limit is given by the spectral density of the corresponding fluctuating parameter. The difference is that in the case of the off-diagonal coupling fluctuations the spectral density is taken at zero frequency, whereas in the case of the energy level fluctuations it is taken at the Rabi frequency.

In the case of the strong-coupling limit, however, the decoherence rate is not related to the fluctuation spectrum. Moreover we found that the electron in the qubit is localized in this limit due to an effective decrease of the off-diagonal coupling. This phenomenon may resemble the localization in the spin-boson model in the strong-coupling limit.

If the charge correlator and the total SET current are affected by the qubit (backaction effect), we found that the off-diagonal density-matrix elements of the qubit survive in the steady-state limit and therefore the relaxation rate is not zero. We concentrated on the case of weak coupling, when the Coulomb repulsion between the qubit and the SET is smaller than the Rabi frequency. The backaction of the qubit on the SET, however, can be weak or strong. In the first case we found that the total damping rate of the qubit due to decoherence and relaxation is again given by the spectral density of the SET charge fluctuations, *modulated by the qubit*. This relation, however, is not working if the backaction is strong. Indeed, we found that the damping rate of the qubit in this case is larger by an order of magnitude than that given by the spectral density of the corresponding fluctuating parameter.

This looks like that in the strong backaction of the qubit on the SET the major component of decoherence is not coming from the fluctuation spectrum of the qubit's parameters only, but also from the measurement signal of the SET. On the first sight it could agree with an analysis of Ref. 30, suggesting that the decoherence rate contains two components, generated by a measurement and by a pure environment (environmental fluctuations). The latter therefore represents an unavoidable decoherence, generated by any environment. Yet, in a weak-coupling regime such a separation seems to be not working. In this case the damping (decoherence) rate is totally determined by the environment

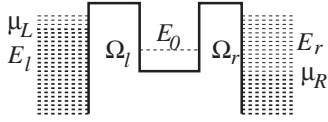


FIG. 10. Resonant tunneling through a single dot.  $\mu_{L,R}$  are the Fermi energies in the collector and emitter, respectively.

fluctuations, even so modulated by the qubit.

Although our model deals with a particular setup, it bears the main physics of a fluctuating environment, acting on a qubit. Indeed, the Bloch-type rate equations, which we used in our analysis, have a pronounced physical meaning: they relate the variation of qubit parameters with a nearby fluctuating field described by rate equations. A particular mechanism that generated this field should not be relevant for an evaluation of the decoherence and relaxation rates, but only its fluctuation spectrum. Indeed, in the weak-coupling limit our result for the decoherence rate coincides with that obtained in a framework of the spin-boson model. Thus our model can be considered as a generic one. Its main advantage is that it can be easily extended to multiple coupled qubits. Such an analysis would allow us to determine how decoherence scales with number of qubits,<sup>42</sup> which is extremely important for a realization of quantum computations.

In addition, our model can be extended to more complicated fluctuating environments, such as containing characteristic frequencies in its spectrum. It would formally correspond to a replacement of the SET in Fig. 2 by a double dot coupled to the reservoirs.<sup>43</sup> All these situations, however, must be a subject of a separate investigation.

#### ACKNOWLEDGMENTS

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#### APPENDIX A: QUANTUM MECHANICAL DERIVATION OF RATE EQUATIONS FOR QUANTUM TRANSPORT

Consider the resonant tunneling through the SET, shown schematically in Fig. 10. The entire system is described by the Hamiltonian  $H_{\text{SET}}$ , given by Eq. (14). The wave function can be written in the same way as Eq. (17), where the variables related to the qubit are omitted,

$$|\Psi(t)\rangle = \left[ b(t) + \sum_l b_{0l}(t) c_0^\dagger c_l + \sum_{l,r} b_{rl}(t) c_r^\dagger c_l + \sum_{l < l',r} b_{0ll'}(t) c_0^\dagger c_r^\dagger c_l c_{l'} + \dots \right] |0\rangle. \quad (\text{A1})$$

Substituting  $|\Psi(t)\rangle$  into the time-dependent Schrödinger equation,  $i\partial_t|\Psi(t)\rangle = H_{\text{SET}}|\Psi(t)\rangle$ , and performing the Laplace

transform,  $\tilde{b}(E) = \int_0^\infty \exp(iEt) b(t) dt$ , we obtain the following infinite set of algebraic equations for the amplitudes  $\tilde{b}(E)$ :

$$E\tilde{b}(E) - \sum_l \Omega_l \tilde{b}_{0l}(E) = i, \quad (\text{A2a})$$

$$(E + E_l - E_0) \tilde{b}_{0l}(E) - \Omega_l \tilde{b}(E) - \sum_r \Omega_r \tilde{b}_{lr}(E) = 0, \quad (\text{A2b})$$

$$(E + E_l - E_r) \tilde{b}_{lr}(E) - \Omega_r \tilde{b}_{0l}(E) - \sum_{l'} \Omega_{l'} \tilde{b}_{0ll'r}(E) = 0, \quad (\text{A2c})$$

$$(E + E_l + E_{l'} - E_0 - E_r) \tilde{b}_{0ll'r}(E) - \Omega_{l'} \tilde{b}_{lr}(E) + \Omega_l \tilde{b}_{l'l'r}(E) - \sum_{r'} \Omega_{r'} \tilde{b}_{ll'r'r'}(E) = 0, \quad (\text{A2d})$$

...

[The right-hand side of Eq. (A2a) reflects the initial condition.]

Let us replace the amplitude  $\tilde{b}$  in the term  $\sum \Omega \tilde{b}$  of each of the Eqs. (A2a)–(A2d) by its expression obtained from the subsequent equation. For example, substituting  $\tilde{b}_{0l}(E)$  from Eq. (A2b) into Eq. (A2a) we obtain

$$\left[ E - \sum_l \frac{\Omega_l^2}{E + E_l - E_0} \right] \tilde{b}(E) - \sum_{l,r} \frac{\Omega_l \Omega_r}{E + E_l - E_0} \tilde{b}_{lr}(E) = i. \quad (\text{A3})$$

Since the states in the reservoirs are very dense (continuum), one can replace the sums over  $l$  and  $r$  by integrals, for instance,  $\sum_l \rightarrow \int \rho_L(E_l) dE_l$ , where  $\rho_L(E_l)$  is the density of states in the emitter, and  $\Omega_{l,r} \rightarrow \Omega_{L,R}(E_{l,r})$ . Consider the first term

$$S_1 = \int_{-\Lambda}^{\mu_L} \frac{\Omega_L^2(E_l)}{E + E_l - E_0} \rho_L(E_l) dE_l, \quad (\text{A4})$$

where  $\Lambda$  is the cut-off parameter. Assuming weak energy dependence of the couplings  $\Omega_{L,R}$  and the density of states  $\rho_{L,R}$ , we find in the limit of high bias,  $\mu_L = \Lambda \rightarrow \infty$ ,

$$S_1 = -i\pi \Omega_L^2(E_0 - E) \rho_L(E_0 - E) = -i \frac{\Gamma_L}{2}. \quad (\text{A5})$$

Consider now the second sum in Eq. (A3).

$$S_2 = \int_{-\Lambda}^{\Lambda} \rho_R(E_r) dE_r \int_{-\Lambda}^{\Lambda} \frac{\Omega_L(E_l) \Omega_R(E_r) \tilde{b}_{lr}(E, E_l, E_r)}{E + E_l - E_0} \rho_L(E_l) dE_l, \quad (\text{A6})$$

where we replaced  $\tilde{b}_{lr}(E)$  by  $\tilde{b}(E, E_l, E_r)$  and took  $\mu_L = \Lambda, \mu_R = -\Lambda$ . In contrast with the first term of Eq. (A3), the amplitude  $\tilde{b}$  is not factorized out the integral (A6). We refer



to this type of terms as “cross terms.” Fortunately, all cross terms vanish in the limit of large bias,  $\Lambda \rightarrow \infty$ . This greatly simplifies the problem and is very crucial for a transformation of the Schrödinger to the rate equations. The reason is that the poles of the integrand in the  $E_l(E_r)$  variable in the cross terms are on the same side of the integration contour. One can find it by using a perturbation series of the amplitudes  $\tilde{b}$  in powers of  $\Omega$ . For instance, from iterations of Eqs. (A2a)–(A2d) one finds

$$\tilde{b}(E, E_l, E_r) = \frac{i\Omega_L\Omega_R}{E(E + E_l - E_r)(E + E_l - E_0)} + \dots \quad (\text{A7})$$

The higher order powers of  $\Omega$  have the same structure. Since  $E \rightarrow E + i\epsilon$  in the Laplace transform, all poles of the amplitude  $\tilde{b}(E, E_l, E_r)$  in the  $E_l$  variable are below the real axis. In this case, substituting Eq. (A7) into Eq. (A6) we find

$$\lim_{\Lambda \rightarrow \infty} \int_{-\Lambda}^{\Lambda} \left[ \frac{\Omega_L\Omega_R}{(E + i\epsilon)(E + E_0 - E_l + i\epsilon)^2(E + E_0 - E_r + i\epsilon)} + \dots \right] dE_l = 0. \quad (\text{A8})$$

Thus,  $S_2 \rightarrow 0$  in the limit of  $\mu_L \rightarrow \infty$ ,  $\mu_R \rightarrow -\infty$ .

Applying analogous considerations to the other equations of the system (A2a)–(A2d), we finally arrive at the following set of equations:

$$(E + i\Gamma_L/2)\tilde{b}(E) = i, \quad (\text{A9a})$$

$$(E + E_l - E_0 + i\Gamma_R/2)\tilde{b}_{0l}(E) - \Omega_l\tilde{b}(E) = 0, \quad (\text{A9b})$$

$$(E + E_l - E_r + i\Gamma_L/2)\tilde{b}_{lr}(E) - \Omega_r\tilde{b}_{0l}(E) = 0, \quad (\text{A9c})$$

$$(E + E_l + E_{l'} - E_0 - E_r + i\Gamma_R/2)\tilde{b}_{0ll'r}(E) - \Omega_{l'}\tilde{b}_{lr}(E) + \Omega_l\tilde{b}_{l'r}(E) = 0, \quad (\text{A9d})$$

...

Equations (A9a)–(A9d) can be transformed directly to the reduced density matrix  $\sigma_{jj'}^{(n,n')}(t)$ , where  $j=0,1$  denote the state of the SET with an unoccupied or occupied dot and  $n$  denotes the number of electrons which have arrived at the collector by time  $t$ . In fact, as follows from our derivation, the diagonal density-matrix elements,  $j=j'$  and  $n=n'$ , form a closed system in the case of resonant tunneling through one level, Fig. 10. The off-diagonal elements,  $j \neq j'$ , appear in the equation of motion whenever more than one discrete level of the system carry the transport [see Eq. (20)]. Therefore we concentrate below on the diagonal density-matrix elements only,  $\sigma_{00}^{(n)}(t) \equiv \sigma_{00}^{(n,n)}(t)$  and  $\sigma_{11}^{(n)}(t) \equiv \sigma_{11}^{(n,n)}(t)$ . Applying the inverse Laplace transform on finds

$$\sigma_{00}^{(n)}(t) = \sum_{l,\dots,r,\dots} \int \frac{dE dE'}{4\pi^2} \tilde{b}_{\underbrace{l,\dots,r,\dots}_n}(E) \tilde{b}_{\underbrace{l,\dots,r,\dots}_n}^*(E') e^{i(E'-E)t}, \quad (\text{A10a})$$

$$\sigma_{11}^{(n)}(t) = \sum_{l,\dots,r,\dots} \int \frac{dE dE'}{4\pi^2} \tilde{b}_{\underbrace{0l,\dots,r,\dots}_{n+1}}(E) \tilde{b}_{\underbrace{0l,\dots,r,\dots}_{n+1}}^*(E') e^{i(E'-E)t}. \quad (\text{A10b})$$

Consider, for instance, the term  $\sigma_{11}^{(0)}(t) = \sum_l |b_{0l}(t)|^2$ . Multiplying Eq. (A9b) by  $\tilde{b}_{0l}^*(E')$  and then subtracting the complex conjugated equation with the interchange  $E \leftrightarrow E'$  we obtain

$$\int \frac{dE dE'}{4\pi^2} \sum_l \left[ (E' - E - i\Gamma_R) \tilde{b}_{0l}(E) \tilde{b}_{0l}^*(E') - 2 \text{Im} \sum_l \Omega_l \tilde{b}_{0l}(E) \tilde{b}^*(E') \right] e^{i(E'-E)t} = 0. \quad (\text{A11})$$

Using Eq. (A10b) one easily finds that the first integral in Eq. (A11) equals to  $-i[\dot{\sigma}_{11}^{(0)}(t) + \Gamma_R \sigma_{11}^{(0)}(t)]$ . Next, substituting

$$\tilde{b}_{0l}(E) = \frac{\Omega_l \tilde{b}(E)}{E + E_l - E_0 + i\Gamma_R/2} \quad (\text{A12})$$

from Eq. (A9b) into the second term of Eq. (A11), and replacing a sum by an integral, one can perform the  $E_l$  integration in the large bias limit,  $\mu_L \rightarrow \infty$ ,  $\mu_R \rightarrow -\infty$ . Then using again Eq. (A10b) one reduces the second term of Eq. (A11) to  $i\Gamma_L \sigma_{00}^{(0)}(t)$ . Finally, Eq. (A11) reads  $\dot{\sigma}_{11}^{(0)}(t) = \Gamma_L \sigma_{00}^{(0)}(t) - \Gamma_R \sigma_{11}^{(0)}(t)$ .

The same algebra can be applied for all other amplitudes  $\tilde{b}_\alpha(t)$ . For instance, by using Eq. (A10a) one easily finds that Eq. (A9c) is converted to the following rate equation:  $\dot{\sigma}_{00}^{(1)}(t) = -\Gamma_L \sigma_{00}^{(1)}(t) + \Gamma_R \sigma_{11}^{(0)}(t)$ . With respect to the states involving more than one electron (hole) in the reservoirs [the amplitudes such as  $\tilde{b}_{0ll'r}(E)$  and so on], the corresponding equations contain the Pauli exchange terms. By converting these equations into those for the density matrix using our procedure, one finds the cross terms, such as  $\sum_l \Omega_l \tilde{b}_{l'r}(E) \Omega_{l'} \tilde{b}_{lr}^*(E')$ , generated by Eq. (A9d). Yet, these terms vanish after an integration over  $E_{l(r)}$  in the large bias limit, as the second term in Eq. (A3). The rest of the algebra remains the same, as described above. Finally, we arrive at the following infinite system of the chain equations for the diagonal elements,  $\sigma_{00}^{(n)}$  and  $\sigma_{11}^{(n)}$ , of the density matrix:

$$\dot{\sigma}_{00}^{(0)}(t) = -\Gamma_L \sigma_{00}^{(0)}(t), \quad (\text{A13a})$$

$$\dot{\sigma}_{11}^{(0)}(t) = \Gamma_L \sigma_{00}^{(0)}(t) - \Gamma_R \sigma_{11}^{(0)}(t), \quad (\text{A13b})$$

$$\dot{\sigma}_{00}^{(1)}(t) = -\Gamma_L \sigma_{00}^{(1)}(t) + \Gamma_R \sigma_{11}^{(0)}(t), \quad (\text{A13c})$$

$$\dot{\sigma}_{11}^{(1)}(t) = \Gamma_L \sigma_{00}^{(1)}(t) - \Gamma_R \sigma_{11}^{(1)}(t), \quad (\text{A13d})$$

...

Summing over  $n$  in Eqs. (A13a)–(A13d) we find for the reduced density matrix of the SET,  $\sigma(t) = \sum_n \sigma^{(n)}(t)$ , the following classical rate equations:

$$\dot{\sigma}_{00}(t) = -\Gamma_L \sigma_{00}(t) + \Gamma_R \sigma_{11}(t), \quad (\text{A14a})$$

$$\dot{\sigma}_{11}(t) = \Gamma_L \sigma_{00}(t) - \Gamma_R \sigma_{11}(t). \quad (\text{A14b})$$

These equations represent a particular case of our general quantum rate equation (20), which is derived using the above described technique.<sup>37,38</sup>

## APPENDIX B: CORRELATOR OF ELECTRIC CHARGE INSIDE THE SINGLE-ELECTRON TRANSISTOR

The charge correlator inside the SET is given by  $S_Q(\omega) = \bar{S}_Q(\omega) + \bar{S}_Q(-\omega)$ , where

$$\bar{S}_Q(\omega) = \int_0^\infty \langle \delta \hat{Q}(0) \delta \hat{Q}(t) \rangle e^{i\omega t} dt. \quad (\text{B1})$$

Here  $\delta \hat{Q}(t) = c_0^\dagger(t) c_0(t) - \bar{q}$  and  $\bar{q} = \bar{P}_1 = P_1(t \rightarrow \infty)$  is the average charge inside the dot. Since the initial state,  $t=0$  in Eq. (B1) corresponds to the steady state, one can represent the time correlator as

$$\langle \delta \hat{Q}(0) \delta \hat{Q}(t) \rangle = \sum_{q=0,1} P_q(0) (q - \bar{q}) (\langle Q_q(t) \rangle - \bar{q}), \quad (\text{B2})$$

where  $P_q(0)$  is the probability of finding the charge  $q=0,1$  inside the quantum dot in the steady state, such that  $P_1(0) = \bar{q}$  and  $P_0(0) = 1 - \bar{q}$ , and  $\langle Q_q(t) \rangle = P_1^{(q)}(t)$  is the average charge in the dot at time  $t$ , starting with the initial condition  $P_1^{(q)}(0) = q$ . Substituting Eq. (B2) into Eq. (B1) we finally obtain

$$\bar{S}_Q(\omega) = \bar{q}(1 - \bar{q}) [\tilde{P}_1^{(1)}(\omega) - \tilde{P}_1^{(0)}(\omega)], \quad (\text{B3})$$

where  $\tilde{P}_1^{(q)}(\omega)$  is a Laplace transform of  $P_1^{(q)}(t)$ . These quantities are obtained directly from the rate equations, such that  $\bar{q} = \bar{\sigma}_{bb} + \bar{\sigma}_{dd}$  and  $\tilde{P}_1^{(q)}(\omega) = \tilde{\sigma}_{bb}^{(q)}(\omega) + \tilde{\sigma}_{dd}^{(q)}(\omega)$ , where  $\bar{\sigma} = \sigma(t \rightarrow \infty)$  and  $\tilde{\sigma}^{(q)}(\omega)$  is the Laplace transform  $\sigma^{(q)}(t)$  with the initial conditions corresponding to the occupied ( $q=1$ ) or

unoccupied ( $q=0$ ) SET. In order to find these quantities it is useful to rewrite the rate equations in the matrix form,  $\dot{\sigma}(t) = M \sigma(t)$ , representing  $\sigma(t)$  as the eight-vector  $\sigma = \{\sigma_{aa}, \sigma_{bb}, \sigma_{cc}, \sigma_{dd}, \sigma_{ac}, \sigma_{ca}, \sigma_{bd}, \sigma_{db}\}$  and  $M$  as the corresponding  $8 \times 8$  matrix. Applying the Laplace transform we find the following matrix equation:

$$(i\omega I + M) \tilde{\sigma}^{(q)}(\omega) = -\sigma^{(q)}(0), \quad (\text{B4})$$

where  $I$  is the unit matrix and  $\sigma^{(q)}(0)$  is the initial condition for the density matrix obtained by projecting the total wave function (17) on occupied ( $q=1$ ) and unoccupied ( $q=0$ ) states of the SET in the limit of  $t \rightarrow \infty$ ,

$$\sigma^{(1)}(0) = \mathcal{N}_1 \{0, \bar{\sigma}_{bb}, 0, \bar{\sigma}_{dd}, 0, 0, \bar{\sigma}_{bd}, \bar{\sigma}_{db}\}, \quad (\text{B5a})$$

$$\sigma^{(0)}(0) = \mathcal{N}_0 \{\bar{\sigma}_{aa}, 0, \bar{\sigma}_{cc}, 0, \bar{\sigma}_{ac}, \bar{\sigma}_{ca}, 0, 0\}, \quad (\text{B5b})$$

and  $\mathcal{N}_1 = 1/\bar{q}$  and  $\mathcal{N}_0 = 1/(1 - \bar{q})$  are the corresponding normalization factors. Finally, one obtains

$$S_Q(\omega) = 2\bar{q}(1 - \bar{q}) \text{Re}[\tilde{\sigma}_{bb}^{(1)}(\omega) + \tilde{\sigma}_{dd}^{(1)}(\omega) - \tilde{\sigma}_{bb}^{(0)}(\omega) - \tilde{\sigma}_{dd}^{(0)}(\omega)]. \quad (\text{B6})$$

In the case shown in Fig. 2 one finds from Eqs. (21a)–(21f) or Eqs. (31a)–(31f) for  $\Gamma'_{L,R} = \Gamma_{L,R}$  that  $\bar{\sigma}_{ac} = \sigma_{bd} = 0$ ,  $\bar{q} = \Gamma_L/\Gamma$ , and  $\tilde{\sigma}_{bb}^{(q)}(\omega) + \tilde{\sigma}_{dd}^{(q)}(\omega) = \tilde{P}_1^{(q)}(\omega)$ . The latter equation is given by

$$(i\omega - \Gamma) \tilde{P}_1^{(q)}(\omega) = -q + \frac{i\Gamma_L}{\omega}. \quad (\text{B7})$$

Substituting Eq. (B7) into Eq. (B3) one obtains

$$S_Q(\omega) = \frac{2\Gamma_L \Gamma_R}{\Gamma(\omega^2 + \Gamma^2)}. \quad (\text{B8})$$

Obviously, for a more general case when  $\Gamma'_{L,R} \neq \Gamma_{L,R}$ , or when the electron-electron interaction excites the electron inside the SET above the Fermi level, Fig. 8, the expressions for  $S_Q(\omega)$ , obtained from Eq. (B6), are more complicated than Eq. (B8).

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